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Papakonstantinou, Athanasios; Pinson, Pierre

Published in:
2014 Proceedings of 13th International Workshop on Large-Scale Integration of Wind Power and Transmission Networks

Publication date:
2014

Document Version
Peer reviewed version

Towards electricity markets accommodating uncertain offers

Athanasios Papakonstantinou and Pierre Pinson
Dept. of Electrical Engineering
Technical University of Denmark
Kgs. Lyngby 2800
athpapa@elektro.dtu.dk ppin@elektro.dtu.dk

***Abstract*—The use of renewable energy sources of energy and in particular wind and solar has been on the rise over the last decades with plans to increase it even more. Such developments introduce significant challenges in existing power systems and can result in high electricity prices and costly infrastructure investments. In this paper we propose a new electricity market mechanism whereby the uncertain and dynamic nature of wind power and other stochastic sources is embedded in the market mechanism itself, by modelling producers' bids as probabilistic estimates. An extension on the bilevel programming formulation of an electricity market, based on the Continuous Ranked Probability Score (CRPS) reduces the impact of poor estimates for both the stochastic producers and the system operator. We introduce a simulation setting which first demonstrates that impact and then proceed to illustrate the main features our market setup and compare it with a conventional electricity market and a standard bilevel setup.**

NOMENCLATURE

i	Index of dispatchable generators in set I
j	Index of stochastic generators in set J
D	Total demand
λ^{shed}	Value of involuntarily shed demand
$\lambda_{Di}, \lambda_{Wj}$	Generators' day-ahead price offers
C_{Di}, C_{Wj}	Generators' capacities
$\lambda^{+}_{Bi}, \lambda^{-}_{Bi}$	Price offers of up and down regulation
R^{+}_{i}, R^{-}_{i}	Available upwards and down regulation
P_i, W_j	Day-ahead market generation
k	Index of balancing scenarios in set $\mathcal{K}$
$W^{0}_{jk}, \widehat{W}^{0}_{jk}$	Actual and forecasted stoch. generation
r^{+}_{ik}, r^{-}_{ik}	Up and down regulation energy in balancing
$W^{\text{spill}}_{jk}, W^{\text{shed}}_{lk}$	Spilled wind and unsupplied load in balancing

I. INTRODUCTION

Electricity is nowadays commonly exchanged through liberalised electricity markets, with a day-ahead mechanism permitting to settle on supply, consumption and prices, and a real-time mechanism for settling on unforeseen deviations from the day-ahead schedule. Such mechanisms were designed in a context where dispatchable generators, with non-negligible marginal costs, were dominating. By depending primarily on conventional (fossil, hydro and nuclear) power generation based on marginal pricing, deterministic market designs were considered adequate with straightforward setups consisting of a forward optimal allocation accompanied by a real-time balancing mechanism.

However, as the stakes of renewable energy increase, such market designs tend to become inefficient since they are not designed to take into account the uncertainty brought by the substantial variability and limited predictability associated with stochastic sources, most notably *wind power* and *solar energy*. In fact for Denmark where 30% of its electricity production is based on wind power, the uncertainty brought by renewable sources directly impacts day-ahead market prices introducing price volatility [1]. In addition to this, high stakes of renewable sources in a day-ahead market which settles independently from the balancing market require significant flexible capacity in order to cope with the forecast errors. It is suggested that if the expected increase in renewables is not met by an increase in flexible capacity balance costs will escalate dramatically [2], [3].

Both challenges can be addressed by introducing a two-stage stochastic programming formulation of the electricity market whereby day-ahead and balancing are jointly cleared so that the day-ahead settlement takes into account the expected real-time costs. In doing so, it is possible to maximise the social welfare while guaranteeing optimal revenues for the individual players. However, current approaches do come with shortcomings. For example, [4] consider the deployment of reserve capacity in their stochastic model. Now although, they promote an energy-only settlement where the capacity is 'converted' to energy through the market mechanism, the participants can still speculate and therefore influence the market clearing. This issue is addressed by [5] who propose a single auction which clears the market and arranges the financial settlement. However, the stochastic clearing requires the flexible generators to accept losses for some wind power production realisations. This issue has been addressed by [6], [7] who propose a stochastic model based on two-level programming (or 'bilevel') which respects the merit-order for all the producers and guarantees revenue adequacy for both day-ahead and balancing markets.

In the context of these recent proposals, the producers are expected to have the opportunity to bid distributions of their production outputs or some summary characteristics (i.e. prediction intervals) along with their prices, instead of just fixed values. Although this type of modelling can be more realistic in capturing the uncertain nature of renewable sources, it brings additional challenges, since the quality of such bids now defines the settlement in the market. It is important to evaluate them in terms of their accuracy, and in order to do so, we propose the use of *strictly proper scoring rules* [8]. Scoring rules were developed for this purpose and have found several applications in evaluating forecasts, from inflation rates and wind resources [9], to information gathered from citizen sensor networks [10]. In terms of energy related applications, to name a few, scoring rules have been used in order to assess the quality of the ensemble forecasts of wind speed issued over Europe [11]

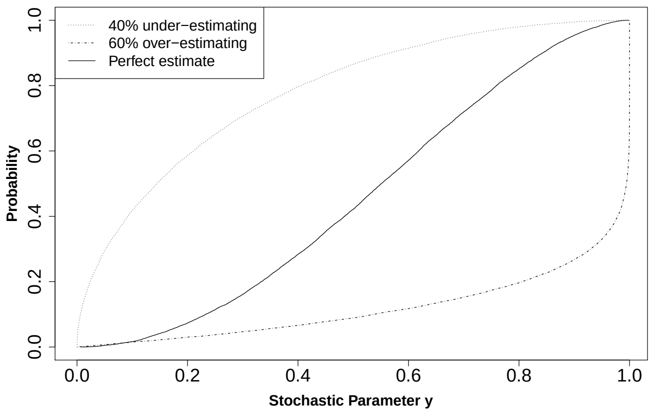


Fig. 1: CDFs for imperfect and perfect estimation

and in order to aggregate reported production levels in virtual power plants [12].

Against this background, we contribute to the state of the art by demonstrating bilevel optimisation's sensitivity on imperfect estimates of stochastic production and by proposing a settlement based on scoring rules where the costs induced by supply uncertainties are reflected on the merit-order definition. We use a simple but informative market setup, and a Monte Carlo simulation of the realisation of a wind producer's output to evaluate the proposed mechanism, benchmarking it against a conventional market design and the standard bilevel model. We show that it is possible to limit the stochastic generator's exposure to balancing costs, and to reduce the market's operation costs.

The rest of the paper is organised as follows: In Section II we describe the setting in more detail, while Section III formally describes the CRPS. Section IV presents the CRPS based bilevel dispatch model and in Section V we introduce the illustrative example, describe the effect of imperfect estimates and compare the various dispatch models. Lastly, Section VI concludes the paper.

II. Uncertainty and Incentives in Electricity Markets

We consider that a wind power producer faces an upper limit C_W in its ouput, defined by the specific technical specifications of the deployed wind farm units and that the real-time generation is equal to yC_W, where $y \in [0, 1]$ is a realisation of the random variable Y which models the wind producer's stochastic behaviour. The variable Y follows a distribution G defined by a set of parameters θ s.t. $Y \sim G(y; \theta)$, where θ can be equal to (μ, σ^2) depending on the definition of the used distribution. A wind power producer will estimate these parameters, and report them to the market operator during the bidding stage of the day-ahead market. We will refer to the distribution that is derived by the estimated parameters $\widehat{\theta}$ as '*estimated distribution*' and denote it by $F(y; \widehat{\theta})$.

Now, as already mentioned, in stochastic programming the market operator uses the stochastic generators' reports to calculate the optimal forward dispatch by estimating real-time balancing based on a number of expected outcomes. These '*balancing scenarios*' represent the market operator's expectation of wind power production and are based on the reported estimated distributions. Naturally, and in consistence to stochastic optimisation literature (cf. [7]) the expected outcomes y_j for each generator follow their respective distributions $F_j(y_j; \widehat{\theta}_j)$ with $j \in \{1, 2, ..., J\}$. However, given the nature of the estimated event and the inevitable errors that are associated with forecasting, these estimates are rarely perfect; it is very common for wind power producers to under or over-estimate their distributions, hence $\widehat{\theta}_j \neq \theta$ with $G \neq F$.

To put this into perspective and in order to numerically evaluate our proposed market setting we will be introducing a specific distribution, without this restricting our theoretical framework in Section IV. In consistence with the related literature (cf. [13], [7]), we will be using a Beta distribution to model the per-unit production of a wind farm. For producer j let W_j^0 be the real-time generation equal to $y_j C_{Wj}$ where $y_j \in (0, 1]$ are realisations of Y_j which follow Beta distributions with parameters $\theta_j = (\mu_j, \sigma_j^2)$ s.t. $Y_j \sim B(a_j, b_j)$. The parameters a and b are derived from (μ, σ^2) as follows:

$$a = \frac{(1-\mu)\mu^2}{\sigma^2} - w, \quad b = \frac{(1-\mu)a}{\mu} \tag{1}$$

Although it is entirely possible that $(\widehat{\mu}_j, \widehat{\sigma}_j^2) \neq (\mu_j, \sigma_j^2)$, with $(\widehat{\mu}_j, \widehat{\sigma}_j^2)$ being the generator j's estimated mean and variance, we simplify our analysis by considering only the estimation of the mean. This translates to $(\widehat{\mu}_j, \widehat{\sigma}_j^2) = (\delta\mu_j, \sigma_j^2)$ with δ being a parameter which denotes the imperfect nature of the estimate. In Figure 1 we demonstrate the differences between the perfect and imperfect estimates by plotting the cumulative distribution function of the Beta distribution with the parameters used in the numerical example in Section V, alongside with the distributions which correspond to over and under estimating of the mean of the actual Beta distribution. As expected, the whole shape of the distribution is affected by mis-estimating the mean, given that as shown in equations (1) the mean influences both a and b.

It becomes clear how important it is to elicit quality predictions of wind power generation within the stochastic optimisation framework as we expect that drawing the balancing scenarios from imperfect estimates will have a severe impact on the final settlement of the market. The above challenge can be addressed through the integration of a strictly proper scoring rule in the proposed market mechanism. As a starting point we will be using the continuous ranked probability score [14] due to its appearance in wind power literature, hence known to be able to model wind forecasts sufficiently[9].

III. Continuous Ranked Probability Score

Before turning to the details of the proposed market dispatch, we formally introduce the CRPS. The CRPS, like all strictly proper scoring rules, incentivises a risk neutral forecaster to truthfully report its forecast by maximizing his expected reward. According to [14], the CRPS is defined as:

$$\text{CRPS} = -\int_{-\infty}^{y} 2F^2(u)du - \int_{y}^{\infty} 2(1 - F(u))^2 du \tag{2}$$

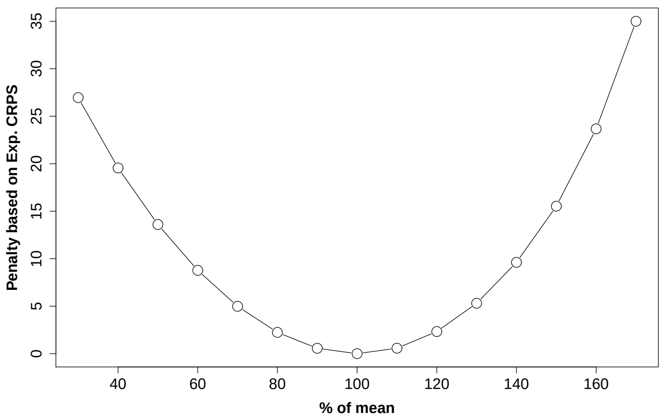


Fig. 2: Expected CRPS penalty

where y is the actual outcome of the forecasted event, and F is the cumulative form of the distribution reported by the forecaster based on parameters $\widehat{\theta}$. Following the calculations in [14], the expected score of a forecaster reporting distribution F, but holding a belief G is:

$$\mathbb{E}[\text{CRPS}] = \qquad (3)$$
$$-\int_{-\infty}^{+\infty}(G(u)-F(u))^2du-\int_{-\infty}^{+\infty}G(u)(1-G(u))du$$

They also prove that CRPS is indeed a strictly proper scoring rule, as the expected score is maximised when the forecaster's report matches his belief (i.e. when $F = G$).

Within the market concept we propose, the CRPS needs to have an impact on the merit-order definition. In order to do so, it needs to be rescaled so that wind producers with severely poor estimates risk being left out from the day-ahead market, while those generating estimates of high quality do not face any repercussions. In this context, the mis-estimation parameter δ is set in a logical interval $[0.3, 1.7]$ s.t. $\widehat{\mu} = [0.3, 0.4, 0.5, ..., 1.5, 1.6, 1.7] \times \mu$ affecting a and b accordingly. Figure 2 describes the rescaled expected CRPS:

$$\mathbb{E}[\text{CRPS}'] =$$
$$\mathcal{S}^{+}\left(1-\frac{\mathbb{E}[\text{CRPS}^{*}]-\min(\mathbb{E}[\text{CRPS}^{*}]}{\max(\mathbb{E}[\text{CRPS}^{*}]-\min(\mathbb{E}[\text{CRPS}^{*}]}\right)+$$
$$\mathcal{S}^{-}\left(\frac{\mathbb{E}[\text{CRPS}^{*}]-\min(\mathbb{E}[\text{CRPS}^{*}]}{\max(\mathbb{E}[\text{CRPS}^{*}]-\min(\mathbb{E}[\text{CRPS}^{*}]}\right) \qquad (4)$$

the minimum and maximum of $\mathbb{E}[\text{CRPS}^{*}]$ is calculated based on the mis-estimation of the beta distributions, with $\text{CRPS}^{*} = -\text{CRPS}$ and parameters $\mathcal{S}^{+}$ and $\mathcal{S}^{-}$ defining the boundaries of the scoring. Since CRPS is expected to penalise agents with imperfect estimates, $\mathcal{S}^{+}$ is set equal to the highest price offer submitted by the dispatchable generators and $\mathcal{S}^{-}$ is set equal to zero.

IV. Dispatch under Uncertainty: Bilevel Optimisation

In this section we introduce our stochastic market settlement model based on bilevel optimisation. Stochastic optimisation methods model more realistically the uncertainty in power systems, by focusing on the interactions between the two stages of the electricity market: day-ahead dispatch and real-time balancing operation. By taking into account the impact of the day-ahead market's clearing on the balancing cost during the real-time market, the overall market efficiency can be improved, especially when considering the deterministic approaches in conventional electricity market setups. However, in some stochastic optimisation models, the flexible producers (i.e. dispatchable producers providing balancing energy in the real-time market) are dispatched out of the merit-order in order to ensure sufficient flexible capability for the balancing. Such settlement, in essence, penalises them for offering balancing service.

On the contrary, the bilevel approach addresses effectively such issues by introducing an additional layer in the decision making process, thus forming a two level optimisation problem. Such formulation is based on the leader-follower strategic game known as Stackelberg game [15] and is naturally relevant to the electricity markets context as it realistically models the interconnection between the day-ahead and balancing stage: The leader (balancing) can optimise its objective based on the optimal respond of the follower (day-ahead market).

In this work we use the bilevel model in [7] as a foundation and further extend it by introducing a CRPS based penalty for the stochastic generators. We maintain the notion that the CRPS penalty affects only the merit order in the day-ahead market dispatch and that it does not impose additional penalties based on the quality of the forecast. This is based on the principle that although it is very important to shield stochastic models from poor estimates of wind power, there should also be minimum external influence on the payments issued by the market settlement as it could potentially discourage investments in renewable energy.

Against this background, given a finite set $\mathcal{K}$ of scenarios of wind power generation $\widehat{W}^{0}_{jk}$ s.t. $\widehat{W}^{0}_{jk} = \widehat{y}_{jk}C_{Wj}$ with y_{jk} sampled from $B(\widehat{a}_j, \widehat{b}_j)$, the objective of the upper-level problem is to compute the optimal value (denoted by $W^{\max}_{j}$) which minimises the day-ahead market dispatch and expected balancing cost in equation (5a) subject to the operation constraints listed in equations (5b)-(5j). The upper-level problem computes the optimal value (denoted by $W^{\max}_{j}$) which minimises the sum of the day-ahead dispatch and expected balancing costs. The lower-level problem, described by equations (5l)-(5o), then optimises the day-ahead market dispatch by setting $W^{\max}_{j}$ as the upper bound for stochastic generator j. The stochastic generators' updated cost $\lambda^{\text{CRPS}'}_{Wj}$ is equal to the generation's marginal cost λ_{Wj} plus the CRPS associated expected penalty $\mathbb{E}[\text{CRPS}']$. Formally, the bilevel model is defined as:

[upper level]: Day-ahead + expected power balancing

$$\text{Min} \sum_{i\in I}\lambda_{Di}P_i+\sum_{j\in J}\lambda^{\text{CRPS}'}_{Wj}W_j+$$
$$\sum_{k\in\mathcal{K}}p_k\Bigg[\sum_{i\in I}\lambda^{+}_{Bi}r^{+}_{ik}-\sum_{i\in I}\lambda^{-}_{Bi}r^{-}_{ik}+\sum_{j\in J}\lambda^{\text{shed}}W^{\text{shed}}_{jk}+$$
$$+\sum_{j\in J}\lambda_{Wj}\left[\widehat{W}^{0}_{jk}-W_j-W^{\text{spill}}_{jk}\right]\Bigg] \qquad (5a)$$

under the following constraints for k-balancing scenarios:

$$\sum_{i \in I} \left[r_{ik}^{+} - r_{ik}^{-}\right] + \sum_{j \in J} W_{jk}^{\text{shed}} + \\ + \sum_{j \in J} \left[\widehat{W}_{jk}^{0} - W_j - W_{jk}^{\text{spill}}\right] = 0 \ : \ \gamma_k^B \tag{5b}$$

$$P_i + r_{ik}^{+} \leq C_{Di} \ \forall i \in I \tag{5c}$$

$$P_i - r_{ik}^{-} \geq 0 \ \forall i \in I \tag{5d}$$

$$r_{ik}^{+} \leq R_i^{+} \ \forall i \in I \tag{5e}$$

$$r_{ik}^{-} \leq R_i^{-} \ \forall i \in I \tag{5f}$$

$$\sum_{j \in J} W_{jk}^{\text{shed}} \leq D \ \forall j \in J \tag{5g}$$

$$W_{jk}^{\text{spill}} \leq \widehat{W}_{jk}^{0} \ \forall j \in J \tag{5h}$$

$$r_{ik}^{+}, r_{ik}^{-}, W_{jk}^{\text{shed}}, W_{jk}^{\text{spill}} \geq 0 \tag{5i}$$

$$0 \leq W_j^{\max} \leq C_{Wj} \ \forall j \in J \tag{5j}$$

[lower level] Day-ahead Market Dispatch

$$\text{Min} \ \sum_{i=1}^{I} \lambda_{Di} P_i + \sum_{j=1}^{J} \lambda_{Wj}^{\text{CRPS}'} W_j \tag{5k}$$

under the following constraints:

$$\sum_{i \in I} P_i + \sum_{j \in J} W_j = D \ : \ \lambda^F \tag{5l}$$

$$P_i \leq C_{Bi} \ \forall i \in I \tag{5m}$$

$$W_j \leq W_j^{\max} (\text{as computed in the upper-level}) \tag{5n}$$

$$P_{Bi}, W_j \geq 0 \ \forall i, j \in I, J \tag{5o}$$

In order to solve the bilevel problem it has to be transformed into an equivalent single level optimisation problem by replacing equations (5k)-(5o) with their Karush-Kuhn-Tucker conditions. Further transformations are required since the KKT complementarity conditions are non-linear. It should be noted that this process is quite technical but well known in the related literature [15], [7], [6], hence we omit it here for conciseness.

V. Numerical Results and Discussion

In this section we introduce a simple market setup which consists of 3 dispatchable generators, with one of them being capable of supplying upwards and downwards balancing, and a single wind power producer (see Table I for the specific variables). Based on the above setup we explore how the standard bilevel model (Improved Dispatch bilevel model in [7]) and its CRPS extension handle imperfect estimates and their impact on the market as over-estimation creates a deficit in production and under-estimation a production surplus.

In this context, we benchmark the stochastic models against a deterministic conventional market setup described by equations (6a)-(6e):

$$\text{Min} \ \sum_{i \in I} \lambda_{Di} P_i + \sum_{j \in J} \lambda_{Wj} W_j \tag{6a}$$

TABLE I: Input Variables in Illustrative Example

Symbol	Value	Symbol	Value
D	170	l	1
λ^{shed}	200	n	1
λ_{W1}	0	C_{Di}	(100,110,50)
λ_{Di}	(35,30,10)	C_{W1}	50
λ_{Bi}^{+}	(40,-,-)	R_i^{+}	(20,-,-)
λ_{Bi}^{-}	(34,-,-)	R_i^{-}	(40,-,-)

under the following constraints:

$$\sum_{i \in I} P_i + \sum_{j \in J} W_j = D \ : \ \lambda_{\text{CON}}^F \tag{6b}$$

$$W_j \leq \widehat{W}_j \ \forall j \in J \tag{6c}$$

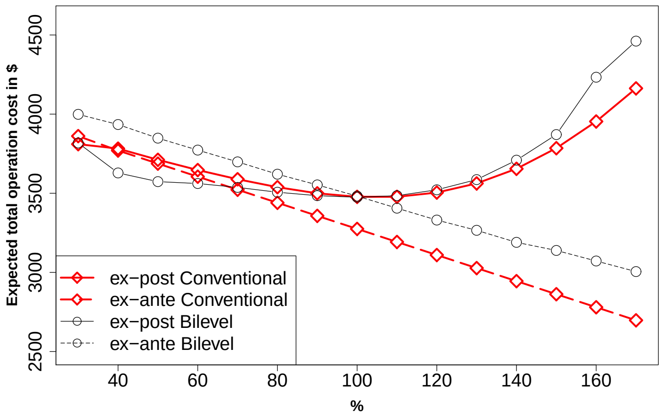
$$P_i \leq C_{Bi} \ \forall i \in I \tag{6d}$$

$$P_{Bi}, W_j \geq 0 \ \forall i, j \in I, J \tag{6e}$$

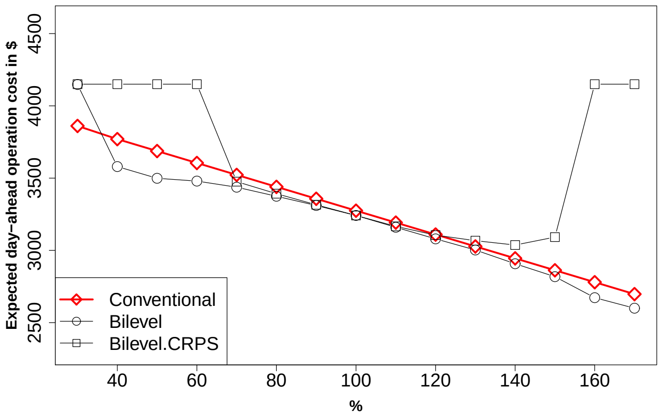
with $\widehat{W}_j$ equal to $\widehat{\mu}_j C_{Wj}$, where $\widehat{\mu}_j$ is the mean of the estimated Beta distribution.

Finally, we simulate the real-time operation by sampling the actual wind power production from the perfectly estimated distribution G, while the balancing scenarios represent the imperfect estimates and are drawn from F. The Monte-Carlo simulation uses a sample of 10^5 realisations, while both bilevel models use 1.5×10^4 balancing scenarios. The analysis consists of two parts: the first part focuses on the market operation costs (i.e. the producers' dispatched power multiplied by their price bid), while the second part focuses on the wind power producer.

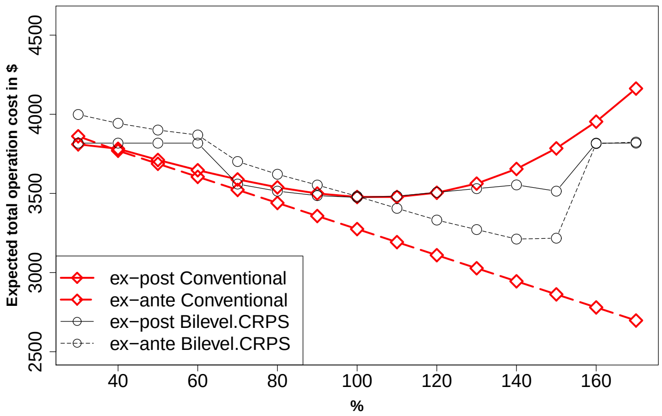
For the first part we introduce two fundamental concepts in stochastic optimisation: a) the ex-ante and b) the ex-post evaluation. In the ex-ante evaluation the solution is computed '*here-and-now*' based solely on the scenarios, hence relies on the possibly imperfect estimates, while the ex-post evaluation reflects on the '*wait-and-see*' approach. In more detail, in the ex-ante evaluation the solution is calculated before the realisation of the wind power production, as opposed to the ex-post valuation which relies on knowledge of the outcome of the estimated event. Moreover, for the ex-ante deterministic conventional model there is no expected balancing and the optimal dispatch is calculated based on the expected value of the wind power production, hence for that model, the total ex-ante operation cost is equal to the ex-post day-ahead operation costs. Now, for all three models, under the ex-post evaluation we compute the optimal balancing profile based on the day-ahead dispatches and a Monte-Carlo simulation of the actual production. Figures 3 and 4 summarise the results of the first stage of the analysis. In particular 3 shows that for the perfect estimate the total market cost is the same for the conventional, standard and CRPS bilevel models (both ex-ante and ex-post evaluations). The equivalence of the ex-ante and ex-post settlements for the bilevel models suggests that the stochastic modelling is robust and that the balancing with 15k scenarios is accurate. However, for imperfect estimates both methods and especially the standard bilevel deteriorate. This is to be expected, since in the ex-ante method balancing relies on the imperfect estimates. Naturally, the gap between ex-ante and ex-post evaluations increases as the quality of the estimate decreases, with this highlighting the negative impact of poor estimates. However, the CRPS bilevel model



(a) Conventional vs. Bilevel

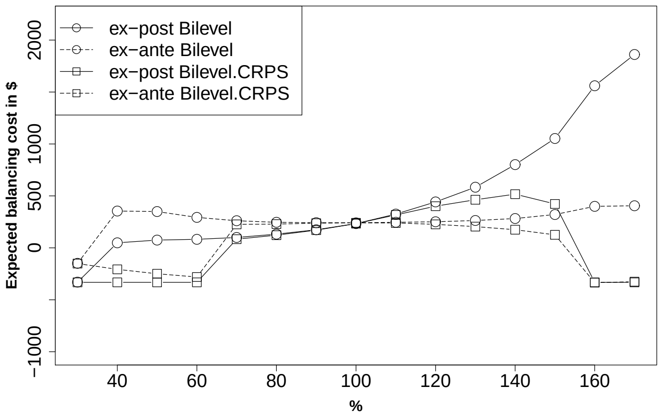


(a) Comparison of all three market models



(b) Conventional vs. Bilevel.CRPS

Fig. 3: Expected total operation costs.



(b) Bilevel vs. Bilevel.CRPS

Fig. 4: Expected day-ahead and balancing operation cost.

is more robust, as the ex-ante evaluation is closer to the ex-post, specially at over-estimating.

An additional positive result is that at severe over-estimating the CRPS bilevel results in lower operation costs when compared to both the conventional deterministic and the standard bilevel models. Figure 4b explains the above result for at least the standard bilevel model as it links over-estimation with heavy balancing costs. Such costs are a result of the expensive load curtailment which happens when the wind power producer exaggerates on its output and creates a deficit which is usually covered by the flexible dispatchable generators (e.g. generator 1). However this may not be the case, as it is possible that the wind power producer pushes the dispatchable generator out of the day-ahead settlement by over-estimating its output. If that generator is the most expensive and happens to be the only offering balancing power (like in our example), then the production deficit is covered by load curtailment.

Nevertheless, the CRPS bilevel model does not result to this type of market behaviour as it can exclude the wind power producer from the day-ahead dispatch, hence shield the system and the wind power producer from the heavy costs. This is further reinforced by the plots in Figure 5 which introduce the second part of the analysis focusing on the wind power producer. We now compute the market revenue the wind power producer expects to derive using the ex-post models and measure the wind power dispatch in the day-ahead markets. The revenue is calculated based on a first-price, energy-only settlement, and is given by:

$$P = \lambda_F W_1 + \mathbb{E}[\lambda_{\mathrm{RT}}^B (W_{1k}^0 - W_1)] \tag{7}$$

where λ_{RT}^B the real-time clearing price, with $\lambda_{\mathrm{RT}}^B = 0$ when wind power is not dispatched in the day-ahead market.

The results in Figure 5 complement those in Figures 4a and 4b and show that the CRPS bilevel model does not dispatch the wind power producer when the estimates are of extremely poor quality. Despite losing the relatively minor gains from under-estimating, this has an overall positive effect on the wind power producer's market revenue, as now it does not face the heavy losses associated with over-estimating due to possible load curtailment in the standard bilevel and conventional models.

Finally, the single-level mixed-integer programming transformation of the bilevel problem (5a)-(5o) has been solved using lp-solve 5.5 under R 3.1.1 on a Linux PC Intel®Core™i7-4770 @ 3.40 GHz with 8GB of RAM. The computational time for the solution of the bilevel problem was 58 minutes, with most of that time spent on the calculation of the sparse matrix of the constraints. We consider this as an upper limit in the required time and since then we have instruments such as the '*value of stochastic solution*' and the '*expected value of perfect information*' will provide useful insights.

Solving such issues will allow us to consider larger networks, closer to a realistic electricity market in terms of both its structure (i.e. multiple buses, transmission constraints etc) and the behaviour of its participants. An important extension linked to the transition to a fully probabilistic market is the consideration of strategic behaviour on behalf of the stochastic producers. Having demonstrated the impact of imperfect estimates the next logical step is to assume some manipulation during and beyond the forecasting stage. This comes natural after the realisation that electricity markets are economic games and as such their participants seek to maximise their gains.

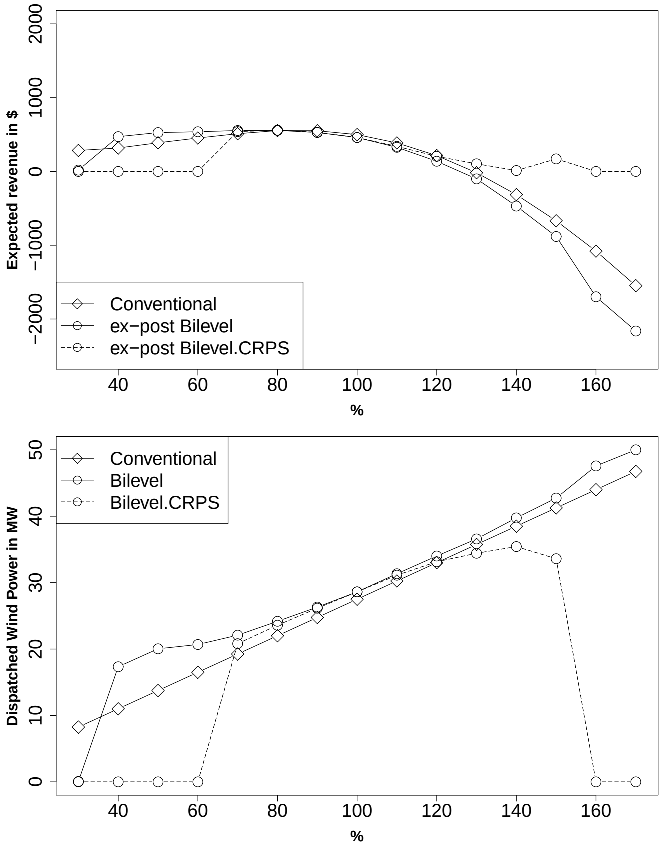


Fig. 5: Market revenue for the wind power producer and dispatch.

considered Gurobi which has reduced computational time to 20 seconds with the same degree of accuracy.

VI. Conclusions

This paper introduces an extension of the bilevel optimisation clearing of an electricity market. It calls for a new approach whereby the stochastic producers' bids are modelled as probabilistic estimates and proposes the use of scoring rules (in particular the CRPS) to evaluate the quality of the bids. The CRPS extension improves the bilevel model as it significantly reduces the gap between the ex-ante and ex-post stochastic evaluations at imperfect estimates. We further show that for cases of severe over-estimation the CRPS bilevel is more economical as it reduces the operation costs by not dispatching wind in the day-ahead market, thus protecting the producers from heavy load curtailment costs.

For future research there are short and long term directions. The short-term research targets involve several technical issues such as the consideration of other strictly proper scoring rules (i.e. a logarithmic one) which can introduce higher penalties to stochastic producers and an alternative scaling of the scoring penalty so it can also capture the effort made during the estimation process based on the principle that those who invest more in generating their predictions should get lower penalties at the event of poor estimates. Also, a formal analysis of the quality of the stochastic solution is needed as the use of 15k balancing scenarios may not be practical in large networks. An analysis based on

Acknowledgement

The authors are partly supported by the Danish Council for Strategic Research (DSF) through the project '5s-Future Electricity Markets', No. 12-132636/DSF. Acknowledgements are due to Stefanos Delikaroglou and Tue V. Jensen for feedback on the code used for the simulations.

References

[1] T. Jonsson, P. Pinson, and H. Madsen, "On the market impact of wind energy forecasts." *Energy Economics*, vol. 32, no. 2, pp. 313–320, 2010.

[2] H. Holttinen, "Impact of hourly wind power variations on the system operation in the nordic countries," *Wind Energy*, vol. 8, pp. 197–218, 2005.

[3] U. Helman, C. Loutan, G. Rosenblum, M. Rothleder, J. Xie, H. Zhou, and M. Kuo, "Integration of renewable resources: Operational requirements and generation fleet capability at 20Rep., 2010.

[4] G. Pritchard, G. Zaker, and A. Philpott, "A single-settlement, energy-only electric power market for unpredictable and intermittent participants." *Operations Research*, vol. 58, pp. 1210–1219, 2010.

[5] J. M. Morales, A. J. Conejo, K. Liu, and J. Zhong, "Pricing electricity in pools with wind producers." *IEEE Transactions On Power Systems*, vol. 27, no. 3, pp. 1366–1376, 2012.

[6] R. Fernandez-Blanco, J. M. Arroyo, and N. Alguacil, "A unified bilevel programming framework for price-based market clearing under marginal pricing," *IEEE Transactions on Power Systems*, vol. 27, no. 1, pp. 517–525, 2012.

[7] J. M. Morales, M. Zugno, S. Pineda, and P. Pinson, "Electricity market clearing with improved scheduling of stochastic production," *European Journal of Operational Research*, vol. 235, no. 3, pp. 765–774, 2014.

[8] A. D. Hendrickson and R. J. Buehler, "Proper scores for probability forecasters," *The Annals of Mathematical Statistics*, vol. 42, no. 6, pp. 1916–1921, 1971.

[9] T. Gneiting and R. Ranjan, "Comparing density forecasts using threshold and quantile weighted proper scoring rules." *Journal of Business and Economic Statistics*, vol. 29, pp. 411–422, 2011.

[10] A. Papakonstantinou, A. Rogers, E. Gerding, and N. Jennings, "Mechanism design for the truthful elicitation of costly probabilistic estimates in distributed information systems," *Artificial Intelligence*, vol. 175, no. 2, pp. 648–672, 2011.

[11] P. Pinson and R. Hagedorn, "Verification of the ECMWF ensemble forecasts of wind speed against analyses and observations." *Meteorological Applications*, vol. 19, no. 4, pp. 484–500, 2012.

[12] V. Robu, R. Kota, G. Chalkiadakis, A. Rogers, and N. Jennings, "Cooperative virtual power plant formation using scoring rules." in *Proceedings of the 26th Conference on Artificial Intelligence*, vol. 3, 2012, pp. 370–376.

[13] X. Liu, "Impact of beta-distributed wind power on economic load dispatch," *Electric Power Components and Systems*, vol. 39, pp. 768–779, 2011.

[14] J. E. Matheson and R. L. Winkler, "Scoring rules for continuous probability distributions," *Management Science*, vol. 22, no. 10, pp. 1087–1096, 1976.

[15] H. von Stackelberg, *Market Structure and Equilibrium.* Springer, 2011.